Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves

Fixed Income

Table of contents

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Calculating a Bond Price given the sequence of Spot Rates

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT+FV}{(1+Z_N)^N} \tag{1}$$

Where:

- Z_1 is the spot rate, or zero-coupon yield or zero rate, for period 1
- Z_2 is the spot rate, or zero-coupon yield or zero rate, for period 2

- Z_N is the spot rate, or zero-coupon yield or zero rate, for period N

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Calculate a Par Rate by solving for PMT

$$100 = \frac{PMT}{(1 + z_1)^1} + \frac{PMT}{(1 + z_2)^2} + \cdots + \frac{PMT + 100}{(1 + z_N)^N} \tag{2}$$

- Equation 2 can be used to calculate a par rate by solving for PMT given a sequence of spot rates $Z_1, Z_2, \ldots, Z_n$.

- This equation is very similar to Equation 1, except $PV = FV = 100$.

- Recall that for a bond to trade at par, its coupon rate and yield-to-maturity must be equal. So, by solving for PMT, we also solve for the yield-to-maturity for the bond to trade at par.

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General formula for the Implied Forward Rate, $IFR_{A,B-A}$

$$(1 + Z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + Z_B)^B \tag{3}$$

- Equation 3 is a general formula for the implied forward rate, $IFR_{A,B-A}$, for a security begins at $t = A$ and matures at $t = B$ (tenor $B - A$)
- To solve for it, we need the spot rate, z_A, and the longer-term spot rate, z_B

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