Learning Module 6: Fixed-Income Bond Valuation: Prices and Yields

Fixed Income

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Present Value (PV) Bond Coupon

$$\text{PV(Bond Coupon)} = \frac{\text{PMT}_t}{(1+r)^t} \tag{1}$$

- The present value (PV) calculation for each bond coupon cash flow (PMT) that occurs in t periods with a market discount rate of r per period should be familiar from an earlier time-value-of-money lesson

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### Present Value (PV) Bond Coupon

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- The present value (PV) calculation for each bond coupon cash flow (PMT)
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Calculating a Bond Price(PV)

$$
PV = \frac{PMT_1}{(1 + r)^1} + \frac{PMT_2}{(1 + r)^2} + \dots + \frac{(PMT_N + FV_N)}{(1 + r)^N} \tag{2}
$$

Equation 2 extends Equation 1 in a general formula for calculating a bond price (PV) given the market discount rate on a coupon date

Where:

- FV is equal to the bond's face value
- N is the number of periods to maturity.

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Yield-to-Maturity (YTM)

- If the market price of a bond is known, Equation 2 can be used to calculate its yield-to-maturity.
- The yield-to-maturity is the internal rate of return on the cash flows—the uniform interest rate such that when the future cash flows are discounted at that rate, the sum of the present values equals the price of the bond. It is an implied or observed single market discount rate.
- The Microsoft Excel or Google Sheets YIELD function may be used for bonds that pay periodic interest:

=YIELD(settlement, maturity, rate, pr, redemption, frequency, [basis])

Where:

- settlement is the settlement date entered using the DATE function
- maturity is the maturity date entered using the DATE function
- rate is the semi-annual (or periodic) coupon
- pr is the price per 100 face value
- redemption is the future value at maturity
- frequency is the number of coupons per year
- [basis] is the day-count convention, typically 0 for US bonds (30/360 day count)

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Full Price

$$PV^{\text{Full}} = PV^{\text{Flat}} + AI \tag{3}$$

- When a bond is priced between coupon payment dates, its price has two components: the flat price (PV^{Flat}) and the accrued interest (AI). The sum of the parts is the full price (PV^{Full}), which also is known as the invoice or "dirty" price

Flat Price

$$\text{Flat Price} = PV^{\text{Full}} - AI$$

- The flat price, which is the full price minus the accrued interest, is also called the quoted or "clean" price and is the type of price usually quoted by bond dealers

- The flat price is used for quotations to avoid misleading investors about a bond's market price trend

- If full prices were quoted by dealers, investors would see price rises each day even if the YTM did not change due to the accrual of interest.

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Accrued Interest

$$
\text{AI} = \frac{t}{T} \times \text{PMT} \tag{4}
$$

- The accrued interest is the portion of the next coupon payment owed to the seller of a bond, because although the seller held the bond for a partial coupon period, the full coupon will be received by the buyer.
- To calculate accrued interest, we determine the fractional amount by counting the days in the period. If the coupon period has T days between payment dates and t days have passed since the last payment, the accrued interest is calculated using Equation 4.

Where:

- t = number of days from the prior coupon payment to the settlement date

- T = number of days in the coupon period
- t/T = fraction of coupon period that has passed since the prior payment
- PMT = coupon payment per period

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Where:

- $t$ = number of days from the prior coupon payment to the settlement date
- $T$ = number of days in the coupon period
- $t/T$ = fraction of coupon period that has passed since the prior payment
- $PMT$ = coupon payment per period
```

Calculate Full Price of a Fixed-Rate Bond

$$PV^{Full} = \frac{PMT}{(1+r)^{1-t/T}} + \frac{PMT}{(1+r)^{2-t/T}} + \cdots + \frac{PMT + FV}{(1+r)^{N-t/T}} \tag{5}$$

- The full price of a fixed-rate bond between coupon payments given the market discount rate per period (r) is the present value of future cash flows as of the trade settlement date, as shown in Equation 5
- This is very similar to Equation 2. The difference is that the next coupon payment (PMT) is discounted for the remainder of the coupon period, which is $1 - t/T$. The second coupon payment is discounted for that fraction plus another full period, $2 - t/T$.

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### Calculate Full Price of a Fixed-Rate Bond

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  another full period, $2 - t/T$.
```

Calculate Full Price of a Fixed-Rate Bond simplified

$$
\begin{aligned}
PV^{Full} &= \left[\frac{PMT}{(1 + r)^1} + \frac{PMT}{(1 + r)^2} + \dots + \frac{PMT + FV}{(1 + r)^N} \right] \times (1 + r)^{t/T} \\
&= PV \times (1 + r)^{t/T} \qquad (6)
\end{aligned}
$$

- Equation 5 is simplified by multiplying both the numerator and denominator by the expression $(1 + r)^{t/T}$. The result is Equation 6

- An advantage of Equation 6 is that the expression in the brackets, PV, is easily obtained using the Excel YIELD function because there are N evenly spaced periods. PV here is the present value of future cash flows as of the last coupon date; it is identical to Equation 2 but is not the same as PV^{Flat}.

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### Calculate Full Price of a Fixed-Rate Bond simplified

$$
P V^{Full} = \left[ \frac{PMT}{(1 + r)^1} + \frac{PMT}{(1 + r)^2} + \dots +
\frac{PMT + FV}{(1 + r)^N} \right] \times (1 + r)^{t/T}
$$
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$$
= PV \times (1 + r)^{t/T} \tag{6}
$$

- Equation 5 is simplified by multiplying both the numerator and denominator
  by the expression $(1 + r)^{t/T}$. The result is Equation 6
- An advantage of Equation 6 is that the expression in the brackets, $PV$, is
  easily obtained using the Excel YIELD function because there are $N$ evenly
  spaced periods. $PV$ here is the present value of future cash flows as of
  the last coupon date; it is identical to Equation 2 but is not the same as
  $PV^{Flat}$.
```