Learning Module 13: Curve-Based and Empirical Fixed-Income Risk Measures

Fixed Income

Table of contents

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Effective Duration (EffDur)

$$EffDur = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta Curve) \times (PV_0)}.$$

- Effective duration and effective convexity are useful for gauging the interest rate risk of bonds whose future cash flows are uncertain.
- Effective duration and effective convexity can be used to estimate the percentage change in a bond's full price for a given shift in the benchmark yield curve.

- Calculating effective duration ($EffDur$) is very similar to calculating approximate modified duration, as shown in Equation 1.
  - The differences are that $\Delta Curve$ is in the denominator—since effective duration is a curve duration statistic, it measures interest rate risk in terms of a parallel shift in the benchmark yield curve—and that PV_{-} and PV_{+} are calculated using option pricing models.
  - These models are covered in more detail in later modules but include such inputs as
    - (1) the length of the call protection period,
    - (2) the schedule of call prices and call dates,
    - (3) an assumption about credit spreads over benchmark yields (which also includes any liquidity spread),
    - (4) an assumption about future interest rate volatility, and
    - (5) the level of market interest rates (e.g., the government par curve). The analyst holds the first four inputs constant and then raises and lowers the fifth input (i.e., parallel shifts) to derive PV_{+} and PV_{-}, respectively.

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Effective Convexity (EffCon)

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- The formula for calculating effective convexity ($EffCon$) is also very similar to the formula for approximate convexity, as shown in Equation 2.

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Estimate the percentage change in a bond’s full price

$$\%\Delta PV^{Full} \approx (-EffDur \times \Delta Curve) + \left[\frac{1}{2} \times EffCon \times (\Delta Curve)^2\right] \tag{3}$$

- Just as with yield-based interest rate risk measures, effective duration and effective convexity can be used to estimate the percentage change in a bond's full price for a given shift in the benchmark yield curve (Δ Curve), as shown in Equation 3.

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Key rate duration (or partial duration)

$$KeyRateDur_k = -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r_k} \tag{4}$$

$$\sum_{k=1}^{n} KeyRateDur_k = EffDur \tag{5}$$

- Key rate duration (or partial duration) is a measure of a bond's sensitivity to a change in the benchmark yield at a specific maturity. Such a measure is important to isolate the price responses of bonds to changes in the rates of key maturities on the benchmark yield curve

- Key rate durations define a security's price sensitivity over a set of maturities along the yield curve, with the sum of key rate durations being equal to the effective duration, as shown in Equation 4 and Equation 5:

Where:

- r_k represents the k th key rate.

- In contrast to effective duration, key rate durations help identify “shaping risk” for a bond—that is, a bond’s sensitivity to changes in the shape of the benchmark yield curve (e.g., the yield curve becoming steeper or flatter or twisting).

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Expected Estimated Price Change

$$\frac{\Delta PV}{PV} = -KeyRateDur_k \times \Delta r_k \tag{6}$$

- Equation 6 rearranges terms from Equation 4 to solve for $\Delta PV/PV$ (or $\%\Delta PV^{Full}$):

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