Time Value of Money

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Future Value

$$FV_t = PV(1+r)^t \tag{1}$$

Where:

- FV_t: future value at time t
- PV: present value at time 0
- r: discount rate per period
- t: number of compounding periods

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```
## Future Value

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FV_t = PV(1+r)^t \tag{1}
$$



Where:
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* $FV_t$: future value at time $t$
* $PV$: present value at time $0$
* $r$: discount rate per period
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```

Future Value with Continuous Compounding

$$
FV_t = PVe^{r t} \tag{2}
$$

Where:

- FV_t: future value at time t
- PV: present value at time 0
- r: discount rate per period
- t: time in continuous periods

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## Future Value with Continuous Compounding

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FV_t = PVe^{r t} \tag{2}
$$

Where:

* $FV_t$: future value at time $t$
* $PV$: present value at time $0$
* $r$: discount rate per period
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```

Present Value

$$FV_t = PV(1+r)^t$$

$$PV = FV_t \left[\frac{1}{(1+r)^t} \right]$$

$$PV = FV_t (1+r)^{-t} \tag{3}$$

Where:

- PV: present value at time 0
- FV_t: future value at time t
- r: discount rate per period
- t: number of compounding periods

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```
## Present Value

$$
FV_t = PV(1+r)^t
$$

$$
PV = FV_t \left[ \frac{1}{(1+r)^t} \right]
$$

$$
PV = FV_t (1+r)^{-t} \tag{3}
$$

Where:

* $PV$: present value at time $0$
* $FV_t$: future value at time $t$
* $r$: discount rate per period
* $t$: number of compounding periods
```

Present Value with Continuous Compounding

$$PV_t = FV e^{-rt} \tag{4}$$

Where:

- PV_t: present value at time t
- FV: future value
- r:
- t:

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```
## Present Value with Continuous Compounding

$$
PV_t = FV e^{-rt} \tag{4}
$$

Where:

* $PV_t$: present value at time $t$
* $FV$: future value
* $r$:
* $t$:
```

Present Value of a Discount (Zero-Coupon) Bond

$$PV(DiscountBond) = \frac{FV_t}{(1+r)^t} \tag{5}$$

Where:

- PV: present value of the bond
- FV_t: principal (face value) paid at maturity

- r: market discount rate per period

- t: number of periods to maturity

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```
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Where:

* $PV$: present value of the bond
* $FV_t$: principal (face value) paid at maturity
* $r$: market discount rate per period
* $t$: number of periods to maturity
```

Present Value of a Coupon Bond

$$
\text{PV(Coupon Bond)}
$$

$$
= \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \cdots + \frac{PMT_N + FV_N}{(1+r)^N} \tag{6}
$$

Where:

- PV: present value of the bond

- PMT_N:
- FV_N: principal repaid at maturity

- r: market discount rate per period

- N: number of periods to maturity

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```
## Present Value of a Coupon Bond

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Where:

* $PV$: present value of the bond
* $PMT_N$:
* $FV_N$: principal repaid at maturity
* $r$: market discount rate per period
* $N$: number of periods to maturity
```

Present Value of a Perpetual Bond (Perpetuity)

$$PV_{\text{Perpetual Bond}} = \frac{PMT}{r} \tag{7}$$

Where:

- PV: present value of the perpetuity
- PMT: fixed periodic payment
- r: discount rate per period

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```
## Present Value of a Perpetual Bond (Perpetuity)

$$
PV_{\text{Perpetual Bond}} = \frac{PMT}{r} \tag{7}
```

```
$$

Where:

* $PV$: present value of the perpetuity
* $PMT$: fixed periodic payment
* $r$: discount rate per period
```

Annuity Payment Formula

$$A = \frac{r(PV)}{1-(1+r)^{-t}} \tag{8}$$

where:

- A = periodic cash flow
- r = market interest rate per period
- PV = present value or principal amount of loan or bond
- t = number of payment periods

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```
## Annuity Payment Formula

$$
A = \frac{r(PV)}{1-(1+r)^{-t}} \tag{8}
$$

where:

* $A$ = periodic cash flow
* $r$ = market interest rate per period
* $PV$ = present value or principal amount of loan or bond
* $t$ = number of payment periods
```

Present Value of Stock with Constant Dividends (Infinite Series)

$$
PV_t = \sum_{i=1}^{\infty} \frac{D_t}{(1+r)^i} \tag{9}
$$

Where:

- PV_t: present value of the stock at time t
- D_t: constant dividend per period
- r:
- i: dividend payment period index

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```
## Present Value of Stock with Constant Dividends (Infinite Series)

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Present Value of Stock with Constant Dividends (Perpetuity Form)

$$
PV_t = \frac{D_t}{r} \tag{10}
$$

Where:

- PV_t: present value of the stock at time t
- D_t: constant dividend per period

- r:

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```
## Present Value of Stock with Constant Dividends (Perpetuity Form)

$$
PV_t = \frac{D_t}{r} \tag{10}
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Where:

* $PV_t$: present value of the stock at time $t$
* $D_t$: constant dividend per period
```

Dividend Growth Formula (One Period Ahead)

$$
D_{t+1} = D_t(1+g) \tag{11}
$$

Where:

- D_{t+1}: dividend at time $t+1$
- D_t: dividend at time t
- g: constant dividend growth rate

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```
## Dividend Growth Formula (One Period Ahead)

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D_{t+1} = D_t(1+g) \tag{11}
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Where:

* $D_{t+1}$: dividend at time $t+1$
* $D_t$: dividend at time $t$
* $g$: constant dividend growth rate
```

Dividend Growth Formula (Multiple Periods)

$$D_{t+i} = D_t(1+g)^i \tag{12}$$

Where:

* D_{t+i}: dividend at time $t+i$
* D_t: dividend at time t
* g: constant dividend growth rate
* i: number of periods after time t

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```
## Dividend Growth Formula (Multiple Periods)

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D_{t+i} = D_t(1+g)^i \tag{12}
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Where:

* $D_{t+i}$: dividend at time $t+i$
* $D_t$: dividend at time $t$
* $g$: constant dividend growth rate
* $i$: number of periods after time $t$
```

Present Value of Stock with Constant Dividend Growth (Infinite Series)

$$PV_t = \sum_{i=1}^{\infty} \frac{D_t(1+g)^i}{(1+r)^i} \tag{13}$$

Where:

* PV_t: present value of the stock at time t

- D_t: dividend at time t
- g: constant dividend growth rate
- r:
- i: dividend payment period index

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```
## Present Value of Stock with Constant Dividend Growth (Infinite Series)

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PV_t = \sum_{i=1}^{\infty} \frac{D_t(1+g)^i}{(1+r)^i} \tag{13}
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Where:

* $PV_t$: present value of the stock at time $t$
* $D_t$: dividend at time $t$
* $g$: constant dividend growth rate
* $r$:
* $i$: dividend payment period index
```

Constant Growth Dividend Discount Model (Gordon Growth Model)

$$
PV_t = \frac{D_t(1+g)}{r-g} = \frac{D_{t+1}}{r-g} \tag{14}
$$

Where:

- PV_t: present value of the stock at time t
- D_t: dividend at time t
- D_{t+1}: dividend at time $t+1$
- r:
- g: constant dividend growth rate

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* $D_t$: dividend at time $t$
* $D_{t+1}$: dividend at time $t+1$
* $r$:
* $g$: constant dividend growth rate
```

---

name? Two-Stage Dividend Discount Model (General Form)

$$
PV_t = \sum_{i=1}^{n} \frac{D_t(1+g_s)^i}{(1+r)^i} + \sum_{j=n+1}^{\infty} \frac{D_{t+n}(1+g_l)^j}{(1+r)^j} \tag{15}
$$

Where:

- PV_t: present value of the stock at time t
- D_t: dividend at time t
- g_s: initial higher short-term dividend growth rate
- g_l: lower long-term dividend growth rate
- r:
- n: number of periods of short-term growth
- i: dividend period index during short-term growth
- j: dividend period index during long-term growth

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```
## name? Two-Stage Dividend Discount Model (General Form)

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PV_t = \sum_{i=1}^{n} \frac{D_t(1+g_s)^i}{(1+r)^i} + \sum_{j=n+1}^{\infty} \frac{D_{t+n}(1+g_
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* $i$: dividend period index during short-term growth
* $j$: dividend period index during long-term growth
```

---

name? Two-Stage Dividend Discount Model (Terminal Value Form)

$$PV_t = \sum_{i=1}^{n} \frac{D_t(1+g_s)^i}{(1+r)^i} + \frac{E(S_{t+n})}{(1+r)^n} \quad (16)$$

Where:

- PV_t: present value of the stock at time t
- D_t: dividend at time t
- g_s: short-term dividend growth rate
- r:
- n: number of periods of short-term growth
- $E(S_{t+n})$: stock value of the stock in n periods

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```
## name? Two-Stage Dividend Discount Model (Terminal Value Form)

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PV_t = \sum_{i=1}^{n} \frac{D_t(1+g_s)^i}{(1+r)^i} + \frac{E(S_{t+n})}{(1+r)^n} \tag{16}
$$

Where:

* $PV_t$: present value of the stock at time $t$
* $D_t$: dividend at time $t$
* $g_s$: short-term dividend growth rate
* $r$:
* $n$: number of periods of short-term growth
* $E(S_{t+n})$: stock value of the stock in $n$ periods
```

Terminal Value

$$
E(S_{t+n}) = \frac{D_{t+n+1}}{r-g_l} \tag{17}
$$

Where:

- $E(S_{t+n})$: stock value of the stock in n periods
- D_{t+n+1}: dividend at time $t+n+1$
- r:
- g_l: long-term dividend growth rate

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```
## Terminal Value

$$
E(S_{t+n}) = \frac{D_{t+n+1}}{r-g_l} \tag{17}
$$



Where:
```

```
* $E(S_{t+n})$: stock value of the stock in $n$ periods
* $D_{t+n+1}$: dividend at time $t+n+1$
* $r$:
* $g_l$: long-term dividend growth rate
```

---

implied periodic return earned over the life of the instrument (t periods)

$$
r = \sqrt[t]{\frac{FV_t}{PV}} - 1 = \left(\frac{FV_t}{PV} \right)^{\frac{1}{t}} - 1 \tag{18}
$$

Where:

- r: implied periodic return earned over the life of the instrument (t periods)
- FV_t: future value at time t
- PV: present value at time 0
- t: number of periods

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```
## implied periodic return earned over the life of the instrument ($t$ periods)

$$
r = \sqrt[t]{\frac{FV_t}{PV}} - 1 = \left( \frac{FV_t}{PV} \right)^{\frac{1}{t}} - 1 \tag{18}
$$

Where:

* $r$: implied periodic return earned over the life of the instrument ($t$ periods)
* $FV_t$: future value at time $t$
* $PV$: present value at time $0$
* $t$: number of periods
```

Yield-to-Maturity Equation for a Coupon Bond

$$
PV(Coupon\ Bond) = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \dots + \frac{PMT_N + FV_N}{(1+r)^N} \tag{19}
$$

Where:

* PV: present value (price) of the bond
* PMT_N: coupon payment at period N
* FV_N: principal repaid at maturity
* r: yield-to-maturity per period
* N: number of periods to maturity

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```
## Yield-to-Maturity Equation for a Coupon Bond

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$$


Where:

* $PV$: present value (price) of the bond
* $PMT_N$: coupon payment at period $N$
* $FV_N$: principal repaid at maturity
* $r$: yield-to-maturity per period
* $N$: number of periods to maturity
```

Dividend Yield under Constant Growth

$$
r - g = \frac{D_t(1+g)}{PV_t} = \frac{D_{t+1}}{PV_t} \tag{20}
$$

Where:

- r: expected or required rate of return?
- g: constant dividend growth rate
- D_{t+1}: dividend expected at time $t+1$
- PV_t: present value of the stock at time t

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```
## Dividend Yield under Constant Growth

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r - g = \frac{D_t(1+g)}{PV_t} = \frac{D_{t+1}}{PV_t} \tag{20}
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Where:

* $r$: expected or required rate of return?
* $g$: constant dividend growth rate
* $D_{t+1}$: dividend expected at time $t+1$
* $PV_t$: present value of the stock at time $t$
```

---

name? implied return on a stock given its expected dividend yield and growth

$$
r = \frac{D_t(1+g)}{PV_t} + g = \frac{D_{t+1}}{PV_t} + g \tag{21}
$$

Where:

- r: expected or required rate of return?
- D_{t+1}: dividend expected at time $t+1$
- PV_t: present value of the stock at time t
- g: constant dividend growth rate

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* $r$: expected or required rate of return?
* $D_{t+1}$: dividend expected at time $t+1$
* $PV_t$: present value of the stock at time $t$
* $g$: constant dividend growth rate
```

Stock's Implied Dividend Growth Rate

$$
g = \frac{r * PV_t - D_t}{PV_t + D_t} = r - \frac{D_{t+1}}{PV_t} \tag{22}
$$

Where:

- g: implied constant dividend growth rate
- r: expected or required rate of return?
- D_{t+1}: dividend expected at time $t+1$
- PV_t: present value of the stock at time t

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```
## Stock's Implied Dividend Growth Rate

$$
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Where:

* $g$: implied constant dividend growth rate
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```

```
* $D_{t+1}$: dividend expected at time $t+1$
* $PV_t$: present value of the stock at time $t$
```

---

name? Price-to-Earnings Ratio

$$
\frac{PV_t}{E_t} = \frac{ \frac{D_t}{E_t} \times (1+g) }{r-g} \tag{23}
$$

Where:

- PV_t: price of the stock at time t
- E_t: earnings per share at time t
- D_t: dividend per share at time t
- r: expected or required rate of return?
- g: constant dividend growth rate

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## name? Price-to-Earnings Ratio

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\frac{PV_t}{E_t} = \frac{ \frac{D_t}{E_t} \times (1+g) }{r-g} \tag{23}
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* $PV_t$: price of the stock at time $t$
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```

Forward Price-to-Earnings Ratio

$$
\frac{PV_t}{E_{t+1}} = \frac{ \frac{D_{t+1}}{E_{t+1}} }{r - g}\tag{24}
$$

Where:

- PV_t: price of the stock at time t
- E_{t+1}: expected earnings per share at time $t+1$
- D_{t+1}: expected dividend per share at time $t+1$
- r: required rate of return
- g: constant dividend growth rate

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```
## Forward Price-to-Earnings Ratio

$$
\frac{PV_t}{E_{t+1}} = \frac{ \frac{D_{t+1}}{E_{t+1}} }{r - g}\tag{24}
$$

Where:

* $PV_t$: price of the stock at time $t$
* $E_{t+1}$: expected earnings per share at time $t+1$
* $D_{t+1}$: expected dividend per share at time $t+1$
* $r$: required rate of return
* $g$: constant dividend growth rate
```

Cash Flow Additivity and Implied Forward Rate Relationship

$$
FV_2 = PV_0 \times (1 + r_2)^2 = PV_0 \times (1 + r_1)(1 + F_{1,1})\tag{25}
$$

Where:

- FV_2: future value in two years

- PV_0: present value at time 0
- r_1: one-year bond rate
- r_2: two-year bond rate
- $F_{1,1}$: the one year forward rate starting in one year

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```
## Cash Flow Additivity and Implied Forward Rate Relationship

$$
FV_2 = PV_0 \times (1+r_2)^2 = PV_0 \times (1+r_1)(1+F_{1,1}) \tag{25}
$$



Where:

* $FV_2$: future value in two years
* $PV_0$: present value at time 0
* $r_1$: one-year bond rate
* $r_2$: two-year bond rate
* $F_{1,1}$: the one year forward rate starting in one year
```

To fix

- confused the r definition for a few formulas wihtout realizing so i question the definition or left it empty
- Some formula names need to be confirmed