Rates and Returns

Quantitative Methods

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Learning Module 3: Rates of Return

Holding Period Return

$$R = \frac{(P_1 - P_0) + I_1}{P_0} \tag{1}$$

Where:

- R: holding period return
- P_1: price at end of period ($t = 1$)
- P_0: price at beginning of period ($t = 0$)
- I_1: income received at end of period

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```
## Holding Period Return

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$$

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* $R$: holding period return
* $P_1$: price at end of period $(t=1)$
* $P_0$: price at beginning of period $(t=0)$
* $I_1$: income received at end of period
```

Arithmetic Mean Return

$$\bar{R}_i = \frac{R_{i1} + R_{i2} + \cdots + R_{i,T-1} + R_{iT}}{T} = \frac{1}{T}\sum_{t=1}^{T} R_{it} \tag{2}$$

Where:

- $\bar{R}_i$: arithmetic mean return for asset i
- R_{it}: return in period t for asset i
- T: total number of periods

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## Arithmetic Mean Return

$$
\bar{R}_i = \frac{R_{i1} + R_{i2} + \cdots + R_{i,T-1} + R_{iT}}{T}
= \frac{1}{T} \sum_{t=1}^{T} R_{it} \tag{2}
$$

Where:

* $\bar{R}_i$: arithmetic mean return for asset $i$
* $R_{it}$: return in period $t$ for asset $i$
* $T$: total number of periods
```

Geometric Mean Return

$$\bar{R}_{Gi} = \sqrt[T]{(1 + R_{i1}) \times (1 + R_{i2}) \times \cdots \times (1 + R_{i,T-1}) \times (1 + R_{iT})} - 1 \tag{3}$$

$$\bar{R}_{Gi} = \sqrt[T]{\prod_{t=1}^{T} (1 + R_t)} - 1$$

Where:

- $\bar{R}_{Gi}$: geometric mean return for asset i
- R_{it}: return in period t for asset i
- T: total number of periods

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## Geometric Mean Return

$$
\bar{R}_{Gi}
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\tag{3}
$$


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\bar{R}_{Gi} = \sqrt[T]{\prod_{t=1}^{T} (1 + R_{t})} - 1
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Where:

* $\bar{R}_{Gi}$: geometric mean return for asset $i$
* $R_{it}$: return in period $t$ for asset $i$
* $T$: total number of periods
```

Harmonic Mean

$$\bar{X}_H = \frac{n}{\sum_{i=1}^{n} (1/X_i)} \tag{4}$$

Where:

- $\bar{X}_H$: harmonic mean
- X_i: observation i (must be positive)
- n: number of observations

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## Harmonic Mean

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\bar{X}_H = \frac{n}{\sum_{i=1}^{n} (1/X_i)} \tag{4}
$$

Where:

* $\bar{X}_H$: harmonic mean
* $X_i$: observation $i$ (must be positive)
* $n$: number of observations
```

Money-Weighted Return (Internal Rate of Return)

$$\sum_{t=0}^{T} \frac{CF_t}{(1 + \text{IRR})^t} = 0 \tag{5}$$

Where:

- IRR: internal rate of return (money-weighted return)
- CF_t: cash flow at time t
- T: number of periods

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```
## Money-Weighted Return (Internal Rate of Return)

$$
\sum_{t=0}^{T} \frac{CF_t}{(1 + \text{IRR})^t} = 0 \tag{5}
$$

Where:

* $\text{IRR}$: internal rate of return (money-weighted return)
```

```
* $CF_t$: cash flow at time $t$
* $T$: number of periods
```

Time-Weighted Return

$$R_{TW} = [(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_N)]^{1/N} - 1 \tag{6}$$

Where:

- R_{TW}: annualized time-weighted return
- R_i: time-weighted return for year i
- N: number of years

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```
## Time-Weighted Return

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R_{TW} = \left[(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_N)\right]^{1/N} - 1 \ta
$$

Where:

* $R_{TW}$: annualized time-weighted return
* $R_i$: time-weighted return for year $i$
* $N$: number of years
```

Present Value with Non-Annual Compounding

$$PV = FV_N \left(1 + \frac{R_s}{m}\right)^{-mN} \tag{7}$$

Where:

- PV: present value
- FV_N: future value at time N
- R_s: quoted annual interest rate

- m: number of compounding periods per year
- N: number of years

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## Present Value with Non-Annual Compounding

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PV = FV_N \left(1 + \frac{R_s}{m}\right)^{-mN} \tag{7}
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Where:

* $PV$: present value
* $FV_N$: future value at time $N$
* $R_s$: quoted annual interest rate
* $m$: number of compounding periods per year
* $N$: number of years
```

Annualized Return from Period Return

$$R_{annual} = (1 + R_{period})^c - 1 \tag{8}$$

Where:

- R_{annual}: annualized return
- R_{period}: return for the period
- c: number of periods in a year

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```
## Annualized Return from Period Return

$$
R_{annual} = (1 + R_{period})^{c} - 1 \tag{8}
$$

Where:

* $R_{annual}$: annualized return
```

```
* $R_{period}$: return for the period
* $c$: number of periods in a year
```

Converting Returns to Weekly

$$R_{weekly} = (1 + R_{daily})^5 - 1; \quad R_{weekly} = (1 + R_{annual})^{1/52} - 1 \tag{9}$$

Where:

- R_{weekly}: weekly return
- R_{daily}: daily return
- R_{annual}: annual return

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```
## Converting Returns to Weekly

$$
R_{weekly} = (1 + R_{daily})^{5} - 1; \quad
R_{weekly} = (1 + R_{annual})^{1/52} - 1 \tag{9}
$$

Where:

* $R_{weekly}$: weekly return
* $R_{daily}$: daily return
* $R_{annual}$: annual return
```

Continuously Compounded Return (associated with a holding period)

$$r_{t,t+1} = \ln(P_{t+1}/P_t) = \ln(1 + R_{t,t+1}) \tag{10}$$

Where:

- here we are using r to refer specifically to continuously compounded returns
- $r_{t,t+1}$: continuously compounded return from t to $t+1$
- P_{t+1}: price at time $t+1$

- P_t: price at time t
- $R_{t,t+1}$:

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```
## Continuously Compounded Return (associated with a holding period)

$$
r_{t,t+1} = \ln(P_{t+1}/P_t) = \ln(1 + R_{t,t+1}) \tag{10}
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Where:

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* $r_{t,t+1}$: continuously compounded return from $t$ to $t+1$
* $P_{t+1}$: price at time $t+1$
* $P_t$: price at time $t$
* $R_{t,t+1}$:
```

Continuously Compounded Return (Multi-Period)

$$r_{0,T} = \ln(P_T/P_0) \tag{11}$$

Where:

- $r_{0,T}$: continuously compounded return from time 0 to T
- P_T: price at time T
- P_0: price at time 0

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```
## Continuously Compounded Return (Multi-Period)

$$
r_{0,T} = \ln(P_T/P_0) \tag{11}
$$

Where:
```

```
* $r_{0,T}$: continuously compounded return from time $0$ to $T$
* $P_T$: price at time $T$
* $P_0$: price at time $0$
```

Price Relatives Product

$$P_T/P_0 = (P_T/P_{T-1})(P_{T-1}/P_{T-2}) \ldots (P_1/P_0) \tag{12}$$

Where:

- P_T/P_0: product of price relatives
- P_T: price at time T
- P_0: price at time 0
- P_t: price at time t

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```
## Price Relatives Product

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P_T/P_0 = (P_T/P_{T-1})(P_{T-1}/P_{T-2}) \ldots (P_1/P_0) \tag{12}
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Where:

* $P_T/P_0$: product of price relatives
* $P_T$: price at time $T$
* $P_0$: price at time $0$
* $P_t$: price at time $t$
```

Sum of Continuously Compounded Returns

$$r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + ... + r_{0,1} \tag{13}$$

Where:

- $r_{0,T}$: continuously compounded return from time 0 to T
- $r_{T,T+1}$:

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## Sum of Continuously Compounded Returns

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r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \ldots + r_{0,1} \tag{13}
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Where:

* $r_{0,T}$: continuously compounded return from time $0$ to $T$
* $r_{T,T+1}$:
```

Real Return

$$(1 + \text{real return}) = (1 + \text{real risk-free rate})(1 + \text{risk premium}) \tag{14}$$

Where:

- $1 + $ real return: the real risk-free return and the risk premium combined
- real risk-free rate:
- risk premium:

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```
## Real Return

$$
(1 + \text{real return}) = (1 + \text{real risk-free rate})(1 + \text{risk premium}) \tag{14}
$$
```

```
Where:

* $1 + \text{real return}$: the real risk-free return and the risk premium combined
* $\text{real risk-free rate}$:
* $\text{risk premium}$:
```

Leveraged Return

$$R_L = \frac{\text{Portfolio return}}{\text{Portfolio equity}} = \frac{[R_p \times (V_E + V_B) - (V_B \times r_D)]}{V_E} = R_p + \frac{V_B}{V_E}(R_p - r_D) \tag{15}$$

$$R_L = R_p + \frac{V_B}{V_E}(R_p - r_D)$$

Where:

- R_L: leveraged portfolio return
- R_p: total investment return on leveraged portfolio
- V_B: debt or borrowed funds
- V_E: equity of the portfolio
- r_D: borrowing cost on debt

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```
## Leveraged Return

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R_L
= \frac{\text{Portfolio return}}{\text{Portfolio equity}}
= \frac{[R_p \times (V_E + V_B) - (V_B \times r_D)]}{V_E}
= R_p + \frac{V_B}{V_E}(R_p - r_D)
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R_L = R_p + \frac{V_B}{V_E}(R_p - r_D)
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Where:
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* $R_L$: leveraged portfolio return
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* $r_D$: borrowing cost on debt
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